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ELABORATING THE NATURE OF MATHEMATICS ACTIVITY IN MULTIPLE LEARNING SETTINGS

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This paper elaborates a framework which contains five components of the nature of mathematics activity inherent in task which addressed in the document. We applied varies of existed principle of task design in different learning settings from literatures to interact with the framework for checking the validity of the inherency with meta-analysis method. An example of task design is given to argue that a good task should be interpreted properly by multiple learning settings. The results of this study were the five components as the fundamental element of task design such as concept development, exploration, practicing, generalizing and reflection.
Keywords: Task design, Nature of mathematics, multiple learning setting,

Introduction

The view that mathematics is learned effectively becomes essential work nowadays. The different perception of mathematics influences the different ways of teacher approach in teaching. Some believe that mathematics was a static knowledge and usually developed abstractly. Others saw it as dynamic discipline and developed it as result of new discoveries from experimentation and application (Croowhite et al, 1986). However, the development of mathematics requires a deeper understanding of concept and connections of mathematics strands. These could be seen in NCTM standards (2000) which described the mathematics instruction should enable students to know and do. Teachers needed to pay attention on these phenomena developed by NTCM since they are the director of the students learning process. Therefore the idea of research concern of how mathematics activity emerged was developed. Teacher's view of how teaching should take place in the classroom is strongly based on a teacher's understanding of the nature of mathematics (Hersh, 1986). Mathematics is as the human activity (Gravemeijer, 1994) and a way of thinking that involves the solution of shared problem solving. The mathematical objects such as concept, procedure, theories, etc will progressively emerged and develop. Thus, it corresponds to the Piagetian constructivist theory that regards to the mathematical conceptualization or mathematics concept development.

Considering this point, different learning settings developed to give opportunity to students to experience the nature of mathematics in their learning activity. The modern reform documents (NCTM, 1989) advocate a situation that is closer to realistic-fundamental (Dossey, 1992). Those two models indicate the importance of the teachers' conception of the nature of mathematics. Moreover, Schoenfeld (1988) in Dossey (1992) argued about the nature of mathematics perceived by students as result of intricate interaction of cognitive and social factor in the context of schooling. Therefore, the essential feature of the nature of mathematics activity must be considered in the teaching and learning process. There were various types of mathematics' activities embedded in different learning settings such as conjecturing, realistic mathematics education, diagnostic teaching, and thought-revealing activities on models and modeling prospective. Every learning setting has their own task design principle for students. However, there were inherency components among different learning settings. The five components of the nature of mathematics activity proposed in this paper were concept development, exploration, practicing, generalizing and reflection.

In this paper, we aimed at showing the five component of the nature of mathematics activity was sufficient component to lead the task design principle (i.e conjecturing, Realistic Mathematics Education, Diagnostic teaching, thought-revealing activities on models and modeling prospective). Moreover, we would present a good mathematics task design by task designer (Hjalmarson & Diefes-Dux, 2008) which could be interpreted by many different task design principles.

Theoretical Framework

In order to explore the nature of the mathematics activity in the different learning setting, we structured the theoretical framework into two parts: first, we described the principles of different learning settings to be analyzed in the next part and second we defined the good mathematics task regarding the nature of mathematics activity and different learning settings.

The different learning settings would have different task design principle. The concept of principle was essential for teachers' community in order to design task based on their settings. The role of principles was more clearly described by Ruthven, Laborde, Leach and Tiberghien (2009) which generated three layers on the theory of didactical situation framework. Those three-layer structures were grand theory, intermediate framework, and design tool. Moreover, the principle was posited as the intermediate framework between the grand theory and the design process of teaching sequences. Moreover, the principle as the intermediate framework itself should reflect on the limitation on the guiding of grand theory. In designing task, Lin (2012) argued about two fundamental functions of principle task design such as it is clearly related to the learners' roles, learning powers or hypothetical learning trajectory; In the practical function of easily evaluating many similar tasks.

As an example, we described the principle of designing Realistic Mathematics Education learning settings. The RME principles (Van den Heuvel & Wijers, 2005) were elaborated as follows: (1) The *activity principle* which focus on the mathematics as a human activity (Freudenthal, 1971, 1973). (2) The *reality principle* refer to the application of mathematical knowledge which was not only posit in the end of learning process but also in the beginning of it and start with context that can be mathematized. (3) The *level principle* concerns on the various level of understanding that experienced by students. It was reflected on the progressive mathematization. (4) The *intertwinement principle* described as the elaboration or integration of various mathematical domains, for instance numbers and measurement are integrated in the learning process. (5) The *interactivity principle* indicated the collaboration learning among learners where students could share strategies and knowledge to others. (6)The *guidance principle* referred to providing guidance by teacher when students re-invent mathematics.

For another example, the principles of activities on models and modeling perspectives developed by Lesh, Hoover, Hole, Kelly and Post (2000) consisted of six principles: (a) the construction principle, (b) the reality principle, (c) the self-assessment principle, (d) the construct documentation principle, (e) the construct share ability and reusability principle, and (f) the effective prototype principle. Those principles were mediating its design tools of modeling cycle including description, manipulation, translation and verification for sequences of modeling tasks. Moreover, Bell (1993) was also studied some teaching experiments and formulate principles of diagnostic teaching (choose a situation, feedback is required, expose cognitive conflict, intervention by adjusting the degree of challenge and reflection & review).

Regarding the appearance of different learning settings with different task design principles, in the predictive analysis (Liljedahl, Chernoff&Zazkis, 2007), we hypothesized that the good mathematics task should not only fit to one learning setting, but it could be analyzed by some different learning settings. To see the availability of task consider the different learning settings, a soccer ball task by Lee(2010) would be analyzed to see the possibility of the different learning settings outcome in the task.

Methodology

We did investigation on the nature of the mathematics activity based on the preceding literature review. We formulated the components of the nature of mathematics activity that could be inherent in the task design and explore the relation of the components. Furthermore, some learning settings would be analyzed to see the inherency of the nature of mathematics activity in the mathematics tasks. A good mathematics task presented and analyzed with regard to the different learning settings and the nature of mathematics activity. Qualitative methods (Merriam, 1998) were applied to analyze some literatures on the principle task design and a good example mathematics task. The analysis of the principles of some different learning settings and the nature of mathematics activity allow us to identify the relation of the nature of mathematics and its appearance in the different mathematics learning settings

Findings

The five components of the nature of mathematics activity

The students doing mathematics is the trademark of the mathematics *concept development*. Meaning, thinking and process were three fundamental elements in the mathematics learning. Take the meaning of the mathematic structure was very important since students need to understand and make sense the mathematics meaning when they learn it. The mathematics learning was also encouraging the thinking. Scusa(2008) proposed the five process of mathematical thinking consist of problem solving, reasoning and proof, communication, representation and connection. Moreover, reasoning and proof became essential since learners should not only explain the procedure that they posed. Learners were also being able to communicate their thinking about mathematics. Another mathematical thinking was making connection. It means that the learners were encouraged to realize that the mathematical ideas are related. Furthermore, the mathematical ideas or concept could be connected to other subject or real world. The third important element in mathematics learning is the process. For us, the mathematical process included the action in order to achieve the mathematics meaning. However, this processes element was intertwined with the thinking element. The thinking element became subset of the process element. The action of the mathematical process could be the way of selecting tools and computational strategies, experimenting, abstracting, generalizing and reflection.

We believed that the *generalization* takes a significant place within the learning and teaching of mathematics. NCTM (2000) examined students to learn and make mathematical generalization. In mathematics, developing skill of making generalization and making it part of the students' mental disposition or habit of mind in learning and dealing with mathematics (www.math4teaching.com). Furthermore, it had sense that it was the synonym for abstraction. As an example, Mason (1996) described the process of generalizing for the heart of development of algebraic reasoning. In order to generalize the mathematics concept, learners should do *exploration* on the mathematics object. The exploration could be through the contextual situation or explore other mathematics notion so that develop new structures. Besides, for achieving the successful mathematics learning or mathematics proficiency (Killpatrick, 2001), *practices* is essential to do. Lave(1993) and Wenger (1998) in Boaler (2002) claim that learning was a process of engagement with practices. Practicing may provide useful framework for consideration of the different aspect of mathematical work (Boaler, 2002). Furthermore, teaching was conceived as active development knowledge, as the on-going development of careful observation, reflection, analysis, and assessment of students and classrooms, not as a bag of trick and method (Ritchie & Wilson, 2000). From that explanation, we could see that the *observation* and *reflection* became two crucial parts in the mathematics activity. For instance, reflection could be committed as a way to increase the effectiveness of the task and avoid the mistakes for the better teaching and learning. Based on the description above, there were five components found as the nature of mathematics activity that could be inherent in task such as the concept development, exploration, practicing, generalizing, and reflection.

The nature of mathematics activity in different learning settings

The nature of mathematics activity was one of three aspects that needed to be considered in designing lesson and building a curriculum (Bell, 1993). In this part, we would investigate the role of the five components of the nature of mathematics activity proposed in the different learning settings. The investigation was done through the principle on the five different learning settings. The first of mathematics learning setting was Realistic Mathematics Education (RME). The concept development component could be derived from the most of the RME principles. The activity principle would require students learn mathematics within the activity developed, reality principle that inherent in the mathematics activity, the guidance principle that provides guidance by teacher and level principle that encourage the progressive mathematization, those supported the *concept development* of mathematics. Moreover, students were also do exploration in each level of RME learning. Historically, the characteristic of RME was related to the Van Hiele level of learning mathematics (de Lange, 1996). Besides, RME was also suggesting the phenomenological exploration on the use of reality principle that embedded in the context (Freudenthal (1991), Treffers (1991)).

Practicing component was also important for RME's learning. According to De Lange (1995), students were supported to practice making 'free production' and reflect on the strategies they take and anticipate the continuation. Besides, the collaboration learning among learners where students could share their strategies and knowledge to others were also become the practicing activity for students. Another component of the nature of mathematics activity is the generalizing component. In addition of the level of the learning process, RME performed the use of models and its level. Those levels of model were the situational level, referential level, general level and level of formal mathematics. The levels of model in the RME learning support the generalizing process from the situational into formal mathematics. Furthermore, the reflection component of the nature of mathematics activity can be done under the guidance principle by teachers. Teachers should guide the students in doing reflection in the interaction community which is also under the principle of interactivity.

Diagnostic teaching as another learning setting task encouraged the concept development and explores the mathematics start with the choosing the situation that appropriate with students and afterwards adjusting degree of challenges and changes the element of mathematics in order to generalize the mathematics structure. The intensity principle that concern on the repetition and practice the essential element in learning is done in the diagnostics teaching environment. Furthermore, one of the most essential parts was the reflection process that could be done through the prediction and check-mode phase. In line with RME, the five components of the nature of mathematics activities went through the thought revealing activities on model and modeling Perspective (Lesh et al, 2000). For instance, some principles of model and modeling perspectives shared the similar view with RME such as the intention on the model construction principle, ability principle and a reality principle. Moreover, the conjecturing learning setting was also encouraged the concept development start with the purposeful observation principle of the problem creation and solution. Based on the observation activity that was given to students, in conjecturing learning setting, students transformed what they observed and constructed which was new mathematics structures and considered as exploration components. The generalization of the mathematics concept could be acquired from the exploration and observation principles. At last, the conjecturing learning setting provided the opportunity to students to do reflection on their work. From the description above, we could see the interaction between the five components of the natures of mathematics activity and the principle of task design in different mathematics learning settings. It means that the five components as sufficient framework are inherent in the different learning settings. So, a good task should be able to interpret by different learning settings. Furthermore, we presented the components of nature of mathematics activity and the multiple learning settings embedded in one mathematics task example called 'Soccer Ball task design' (Lee, 2010).

The Soccer Ball task design

The Soccer Ball task aimed at developing students' understanding of the underlying structures and the relation among conceptual elements in semi-regular polyhedron using the cultural artifact (physical entity) (Lee, 2010). The activity gave opportunity to students to intend in investigating why making the soccer ball should be with such strict. The descriptions of the main activity in each lesson were as follows:

1. In the first lesson, students were engaged to the activity of observing a virtual soccer ball picture and afterwards construct a soccer ball model from tangible material. Through the lesson, teacher wanted to emphasize the value to increase the students' familiarity with cultural artefact problem creation. By given the virtual picture of the soccer ball and students were intended to investigate why manufacturer make the arrangement of the soccer ball in that way. Moreover, the first lesson of a virtual soccer ball image ensured the bucky ball model (constructed material) composed of faces regular pentagon and hexagon. Students curious and express their interest on the reason of the manufacturers uses the pentagon and hexagon instead of other types of polygon. Therefore students pay

attention on mathematical object such as the vertices, edges, faces and angles of the polygon.

2. The second lesson, teacher asked students to explore an alternative model which is still regular polygons for soccer ball. By using the classmate ideas that pay attention on the vertices, edges, faces and angles, students tried to explore the direction counting systematically by counted the number of hexagon and pentagon that connected to hexagon. Moreover, exploring the possibility on making ball like solid using different polygon and find the number of ball-like solids become students' activity.
3. The third lesson was the centre on reflection. Students are given opportunity to reflect on the modelling process and constructing buckyball and created meanings and argumentation. Teacher tried to guide students by informing that the students' argument should not only focus on semi regular polyhedron investigated in classroom but in general. Students tended to use different strategies and some failed to construct buckyball model. The failure of crafting the buckyball model was the reason to pay attention on the arrangement of polygons at each vertex. Teacher guided students narrowing the scope by giving questions about the number of ball-like solids using different polygon. Some students try to examine why manufacturer prefer to use (5,6,6) models named for one pentagon and two hexagons for every vertex. Moreover, the sum of interior angles at each vertex calculated and compared with 360^o. The sum of angles at each vertex should be approximately 360^o. Students created table of the sum of interior angle and find the general properties or patterns from the comparison among the solids. Furthermore, students did kicking and throwing of the buckyball as the reflection and evaluation of their construction.

The Nature of Mathematics activity in the soccer ball task

In the three lessons above, there were some concept that tried to be developed by students such as exploring the pseudo-mathematical object through pseudo sphere, the combinatorial content by each direction and division of angle around the vertices. Besides, the concept development was also interpreted by the symbolic representation of "cell" or "unit" to express the model move from operation on physical object to mental object. Hence, concept development which was one of the components of the nature of mathematics learning appears in the soccer ball lesson design. Those entire concepts are new for students. They tried to do the conjecturing in order to achieve the mathematics concept.

The exploration of the mathematics concept could also be seen from the process on transforming perceptual motion or translating the kicking of the motion (the feeling of kicking) to the division angle sum around each vertex. In other words, the translation of the degree of feeling into mathematics object was done in order to explore the mathematics. By that activity, students are practicing their degree of feeling of motion of bucky ball. Moreover, students in the lesson tried to search alternative model of pseudo sphere using constructed material.

The generalizing process could be seen from the activity of creating table of the angle sum of interior angle and get the pattern. By seeing the pattern, students conjectured that the sum of angles jointed at each vertex of (5, 6, 6) model as virtual model was most similar to a sphere and the number of faces were not too many compare to others. Students said that they will have a great difficulty for manufacturers if they should sew too many faces and higher cost will also be applied. The reflection part could be derived from observation and construction technique throughout the three lesson period. Moreover, teacher also invited students to do reflection by investigating the construct physically (kicking and throwing of bucky ball) and see whether it is rolled well or not. From the description above, we can see that *the five components of the nature of mathematics activity inherent in the soccer ball task design.*

Modeling and conjecturing in Soccer Ball task design

In addition, we would analyze the soccer ball activity with different learning setting. For instance, the conjecturing learning setting with observation, construction, transformation and reflection principles would be explored in the soccer ball activity. The observation was inherent in the activity since the beginning. Observation activity by students happens when they observe the virtual soccer ball image and the mathematics properties in the pseudo sphere construction. The principle construction encouraged opportunities to construct new definition, concept problems, theorems and representation systems (Lesh&Doerr, 2003). Therefore, the new construction of the mathematics and representation was in constructing bucky ball. Students were also predicting the possible arrangement of the bucky ball so that it could roll well. Furthermora, transformation became the essential principles in the activity. The transformation could be seen from the behavior of students in kicking and tossed their construct to investigate whether they make effective soccer ball or not and translated it into mathematics description. Furthermore, reflection on observation and construction were the main technique by teacher throughout the lesson design. It prompted students to analyze their conjectures of the generalization of mathematics object through modeling and induction process.

The thought revealing activities on model and modeling perspectives principle are also inherent in the soccer ball task. The modeling principle can be seen from the activity of modeling the virtual soccer ball into bucky ball and constructing model documentation at the creating a table as generalization of mathematics concept of angle sum of interior angle. As reflection in the learning process, there was a self-assessment principle by students to make sure their construction of the bucky ball. They are supported to have the motion feeling of throwing the bucky ball to make sure the arrangement of the polygon is correct and appropriate.

RME in Soccer Ball task design

The soccer ball task design provides some activities for students. The activity starts with realistic context such as the soccer ball context that familiar for students that reflect to the reality principle of designing RME task. Students were mathematizing the division of angle around the vertices through the observation of the construction of the polygon's construction. The mathematical symbols and models in the soccer ball activity were developed via bottom-up manner (Lee, 2010). In other words, the concept development was passing some levels. Moreover, there were two kinds of models that are considered in the RME learning process such as'model of' and 'model for'. The virtual soccer ball picture became the model of the next mathematics activity which constructing soccer using tangible material. By observing the virtual picture, students were supported to construct their own soccer ball called 'bucky ball'. The constructed bucky ball later became the 'model of' mathematical reasoning which students can pose their understanding of geometry properties in their construction and reasoning by throwing the bucky ball to see the effectiveness of their construction. This was also part of horizontal mathematizing in RME that transform problem field into mathematical problem. Besides, the intertwinement principle inherent in the soccer ball activity. The geometrical object and the division of angle around the vertices and the angle sum of interior angle were learned by students. Moreover, the interactivity principle that let the collaboration learning among students with the guidance from teacher is also described in the learning process situation of soccer ball.

The Diagnostic teaching principle (Bell, 1993) in the Soccer Ball task design

The choses situation as the starting point in the task design was applied in the soccer ball task design. The situation chosen was closed to the students' world. Within the context, students were proposed to the learners which bring into play the concept and relation. Students were also intense in tring to construct the bucky ball model. The degree of challenges of the mathematics in the soccer ball task design was devoted by teacher by pose guidance question and instruction of elaborating the number of different polygon in constructing ball-like solid. Hence, Reflection and review become one of the key principles in diagnostic teaching that also important phase in this task.

Conclusion

We have proposed the five components of the nature of mathematics activity such as concept development, exploration, practicing, generalizing and reflection. The inherency of those five components could be situated in different learning settings: diagnostic teaching, RME, thought revealing activities on model and modeling perspectives and conjecturing learning setting. Hence, the good task design should be able to cross over different learning settings which mean it has the sense of those five components and those five components can be sufficient framework in designing mathematics task. Furthermore the multiple learning settings can be interpreted in the Soccer Ball task design and we could claim it as example of good task design.

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